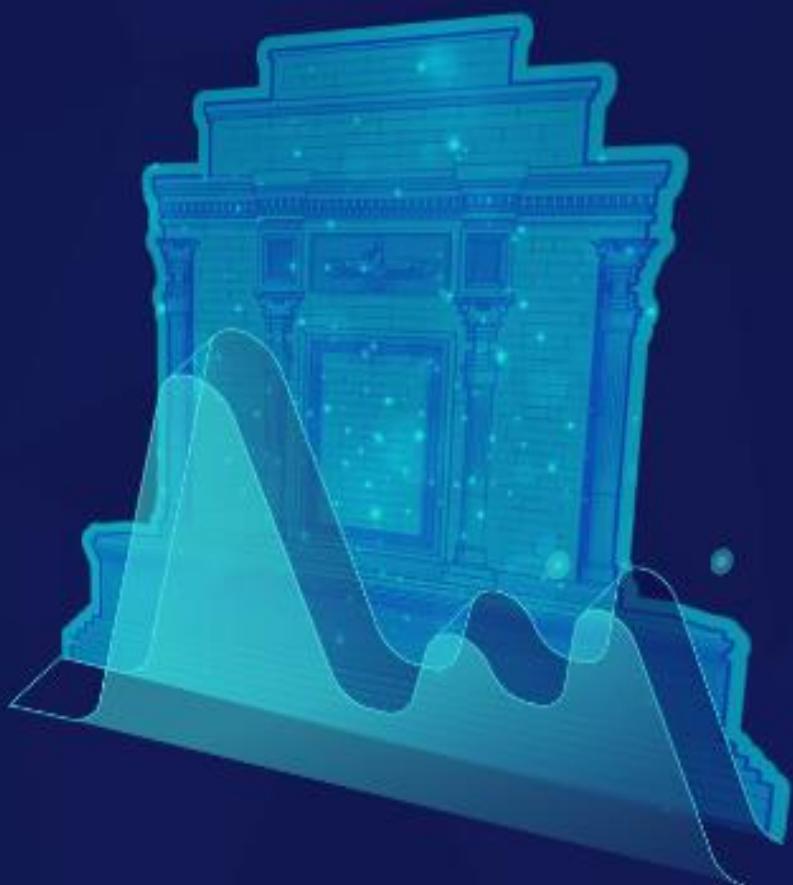


خلاصه مقالات

۲۴ امین کارگاه فرآیندهای تصادفی کاربردی



۱۴۰۱-۳-اسفند ماه
دانشگاه فردوسی مشهد

نام اثر:	خلاصه مقالات ۲۴ امین کارگاه فرآیندهای تصادفی کاربردی
گردآورنده:	سیده طیبه جعفری پور
ناشر:	کمیته برگزاری کارگاه

۲۴ امین کارگاه فرآیندهای تصادفی کاربردی
۱۴۰۱۳ و ۱۴۰۱۴ اسفند

دبير سمینار :

• دکتر وحید فکور

دبير کمیته علمی:

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- دکتر بزرگ نیا
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- دکتر پرورده
- مسعود کوثری
- دکتر زارع پور
- سیده طیبه جعفری پور
- دکتر زنگنه
- دکتر سلطانی
- دکتر علیشاھی
- دکتر فکور
- Dr.Xiao -

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سخن دبیر کارگاه

با یاری خداوند متعال، ۲۴ امین کارگاه فرایندهای تصادفی کاربردی در روزهای سوم و چهارم اسفند ماه سال ۱۴۰۱ در دانشگاه فردوسی مشهد برگزار شد. از جمله اهداف این کارگاه، گرد هم آوردن محققانی است که در زمینه فرآیندهای تصادفی و کاربردهای آن پژوهش و تحقیق دارند. آشنایی با مطالب به روز این دانش و همچنین ایجاد محفلی برای بحث و مناظره در مورد کاربرد فرآیندهای تصادفی و روش های مورد استفاده در آن از دیگر اهداف این کارگاه هستند.

از موسس این سلسله کارگاهها، پروفسور سلطانی، استاد برجسته آمار دانشگاه شیراز، تشکر فراوان دارم که با ایجاد این کارگاه، ما را در اینجا گرد هم آورده تا یافته هایمان را با یکدیگر به اشتراک بگذاریم. همچنین از همه اعضای کمیته علمی و اجرایی که بدون آنها برگزاری کارگاه میسر نبود، تشکر فراوان دارم. از همه نویسندها و شرکت کنندگان برای حضور و ارائه تخصص خود سپاسگزارم.

افتخار نصیب ما گردید تا همراه با برگزاری ۲۴ امین کارگاه فرایندهای تصادفی کاربردی، نکوداشتی برای آقای دکتر ابوالقاسم میامئی، استاد فرهیخته ریاضی و زاده خراسان بزرگ داشته باشیم. با آرزوی سلامتی برای استاد.

وحید فکور

دبیر ۲۴ امین کارگاه فرایندهای تصادفی کاربردی

اسفند ۱۴۰۱

جدول برنامه‌ها

Wednesday, 22nd Feb.

Chairman	Time ¹	Title	Presenter	Affiliation
	8:30 – 9:00	Welcome		
G.R. Mohtashami	9:00 — 10:00	On Discrete Distributions Induced by Certain Discrete Renewel Processes	A.R. Soltani *	Kuwait University
	10:00-10:15	Break		
	10:15-10:45	رگرسیون لوزیستیک فازی تحت خطاهای تصادفی فازی	H. Nili *	University of Birjand
	10:45—11:45	روایت سفری پر ماجرا: گریزانی از احتمال تا علاقه مندی و افر به نظریه احتمال و فرایندهای تصادفی	B. Zangeneh	Sharif University of Technology
		Visiting the university museum		
V. Fakoor	15:00 — 16:00	Prevalent Cohort Studies: Length-Biased Sampling with Right Censoring	M. Asgharian	McGill University
	16:00—16:50	An Improved Lower Bound on the Largest Common Subtree of Random Leaf-Labeled Binary Trees	A. Khezeli	INRIA Paris and Tarbiat Modares University
		Break		
A.R. Soltani	17:00—18:00	Bayesian nonparametric prior via negative binomial process	M. Zarepour	Ottawa University
	18:00—19:00	Local Times of Gaussian Random Fields	Y. Xiao	Mishigan State University

1- All times are in local time.

*-Lectures are held in person.

2- Lecture link: <http://vroom.um.ac.ir/fakoor>

Thursday, 23rd Feb.

Chairman	Time ¹	Title	Presenter	Affiliation
	8:00 - 8:30	Asymptotic Behaviour of the Hazard Function from Cross-sectional Cohort Survival Data	A. Shariati	Macquarie University
V. Fakoor	8:30 – 9:30	Flexible Nonparametric Curve Estimation by Tilting	H. Doosti	Macquarie University
	9:30—10:00	Modified Principal Points	H. Mardani-Fard	Yasuj University
		Break		
Dr. Zamani	10:10 – 10:40	Option valuation in markets with finite liquidity under fractional CEV assets	A.Ghasemifard	University of Mazandaran
	10:40 – 11:10	Fractional Browning motion in financial markets and greeks by malliavin calculus	M. Tahmasbi	Tarbiat Modares University
	11:10—11:40	An Agent-Based Model for Limit Order Markets and its Application in Price Manipulation Detection	E. Salavati	Amir Kabir University Of technology
	11:40—12:10	بیمه سبد سرمایه تحت مدل‌های رژیم-متغیر مارکوف	A.Forush Bastani	I.A.S.B.S.
		Break		
V. Fakoor	15:00 –16:00	Stein method meets algebra	E. Azmoodeh	Liverpool University
	16:00 – 16:50	A new Dimensionality Reduction Method for Multivariate Time Series Analysis	Y. Samadi	Southern Illinois University
		Break		
A.R.Soltani	17:00 –18:00	The van Dantzig problem and the Riemann hypothesis	P. Patie	Cornell University
	18:00 – 19:00	Nonparametric Statistical Inference for i.i.d. Sparsely Observed Diffusions: an FDA Perspective	N. Mohammadi	Colorado State University
	19:00-20:00	Vlasov-McKean Equations	S.Mehri	University Of Warwick
		Closing		

1- All times are in local time.

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رگرسیون لوزستیک فازی تحت خطاهای تصادفی فازی

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چکیده مقالات

انگلیسی

Prevalent cohort studies: Length-Biased sampling with right censoring

M. Asgharian

Department of Mathematics and Statistics, McGill University

Logistic or other constraints often preclude the possibility of conducting incident cohort studies. A feasible alternative in such cases is to conduct a cross-sectional prevalent cohort study for which we recruit prevalent cases, that is, subjects who have already experienced the initiating event, say the onset of a disease. When the interest lies in estimating the lifespan between the initiating event and a terminating event, say death for instance, such subjects may be followed prospectively until the terminating event or loss to follow-up, whichever happens first. It is well known that prevalent cases have, on average, longer lifespans. As such, they do not form a random sample from the target population; they comprise a biased sample. If the initiating events are generated from a stationary Poisson process, the so-called stationarity assumption, this bias is called length bias. I present the basics of nonparametric inference using length-biased right censored failure time data. I'll then discuss some recent progress and current challenges. Our study is mainly motivated by challenges and questions raised in analyzing survival data collected on patients with dementia as part of a nationwide study in Canada, called the Canadian Study of Health and Aging (CSHA). I'll use these data throughout the talk to discuss and motivate our methodologies and their applications.

Stein method meets algebra

E. Azmoodeh

Department of Mathematical Sciences, Liverpool University

Stein method is a three-step program to measure distance between a generic probability measure of that a given target probability distribution. The first step is to find a suitable (differential) Stein operator that characterises the target distribution. In this talk, we introduce the novel notion of an algebraic polynomial Stein operator and show that every polynomial Stein operator of a Gaussian polynomial target distribution in dimension one is algebraic. This opens a new door to find algorithmically Stein operators of the complex target distributions in higher Wiener chaoses. We also discuss the class PSO(N) of the polynomial Stein operators associated to the standard Gaussian distribution and how it connects Stein method to non-commutative algebra. The talk is based on a series of joint works with Dario Gasbarra (Helsinki) and Robert Gaunt (Manchester).

The van Dantzig problem and the Riemann hypothesis

P. Patie

School of Operations Research and Information Engineering (ORIE), Cornell University

In this talk, we start by introducing the intriguing van Dantzig problem which consists in characterizing the subset of Fourier transforms of probability measures on the real line that remain invariant under the composition of two involutions.

We first focus on the so-called Lukacs class of solutions that is the ones that belong to the set of Laguerre-Pólya functions which are entire functions with only real zeros. In particular, we show that the Riemann hypothesis is equivalent to the membership to the Lukacs class of the Riemann ξ function.

We state several closure properties of this class including adaptation of known results of Pólya, de Bruijn and Newman but also some new ones. We proceed by presenting a new class of entire functions, which is in bijection with a set of continuous negative definite functions, that are solutions to the van Dantzig problem and discuss the possibility of the Riemann ξ function to belong to this class.

An improved lower bound on the largest common subtree of random leaf-labeled binary trees

A. Khezeli

INRIA Paris, Tarbiat Modares University

In some models of random combinatorial structures, an object of interest is the largest common substructures of two samples. A well-studied example is the size of the largest increasing subsequence in a random permutation, which can be regarded as the largest common subset in which the permutation induces the same total order as the identity permutation (or another random permutation). In the context of trees, the largest common subtree has been proposed in biology as a measure of similarity between two phylogenetic trees (also known as the maximum agreement subtree). With this motivations, some works have studied the expected size of the largest common subtree of two independent random trees. In this paper, we consider two random binary trees built on n common labeled leaves. It is known that the expected size of the largest common subtree is of order between $n^{0.366}$ and $n^{1/2}$. We improve the lower bound to order $n^{0.4464}$ by constructing a common subtree recursively and by proving a lower bound for its asymptotic growth. The construction is a modification of an algorithm proposed by D. Aldous by splitting the tree at the centroid and by proceeding recursively.

Flexible nonparametric curve estimation by tilting

H. Doosti

School of Mathematical and Physical Sciences, Macquarie University

We wish to highlight two extensions of the results of [3,4] in nonparametric curve estimation. The first extension is using the tilting technique in nonparametric regression function estimation [2] and the second one is its application in nonparametric additive models [1]. [3,4] introduced new high-order, non-parametric density estimators based on data perturbation, e.g. by tilting or data sharpening. The estimators, proposed in [3], produce more accurate estimations than the high-order methods because they remove those negative parts of the estimator which always penalise performance.

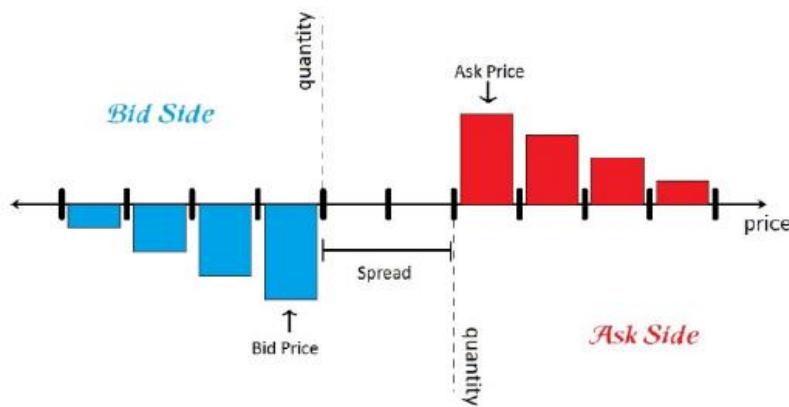
An agent-based model for limit order markets and its Application in price manipulation detection

M. Zare, O. Naghshineh, A. Mohammadpour, E. Salavati*

Department of Mathematics and Computer Science, Amirkabir University of Technology

Almost all financial markets are governed by limit order books (LOBs). LOBs are queue systems that record the outstanding buy and sell orders in the market, and perform their execution or canceling based on the rules of the market. The study of LOBs makes the field of market micro-structure.

The analysis of LOB data is more challenging than classical financial datasets such as price time-series, because of the specific two-dimensional structure of them. Indeed, at each time step the data consists of a set of buy orders and sell orders at different prices with different amounts. The following picture shows a snapshot of an LOB,



We provide a stochastic model for LOBs based on Poisson process, and derive estimations for the parameters of the model. Our model belongs to class of agent-based models, in the sense that it independently models each participant in the market. We use our model for the study of market manipulation phenomena, which is the activity of some traders in the market who send orders with the purpose of influencing the price to their own benefit.

* The main speaker

Bayesian nonparametric prior via negative binomial process

M. Zarepour

Department of Mathematics and Statistics, Ottawa University

We review some simple properties of the Poisson random measure with Lebesgue mean measure λ , $\text{PRM}(\lambda)$. We then study some important functionals of this process. A certain functional of $\text{PRM}(\lambda)$ that stands out in this case, is the negative binomial process (NBP). Using a new representation for NBP, we define a generalized Poisson-Kingman distribution and its corresponding random discrete probability measure. This random discrete probability measure provides a new set of priors with more flexibility in application in nonparametric Bayesian models. We then show how this random discrete probability measure relates to the well-known nonparametric Bayesian priors such as Dirichlet process, normalized positive α -stable process with index in $(0,1)$, Poisson-Dirichlet process, normalized generalized gamma process, etc. Also, using our proposed representation for NBP, we derive a new series representation for the Poisson-Dirichlet process. We also introduce an approximation based on this new representation for the Poisson-Dirichlet process.



On discrete distributions induced by certain discrete renewal processes

A. R. Soltani

**Department of Statistics and Operations Research, College of Science, Kuwait
University**

Firstly, we establish recursive formulas for computing and formulating the probability mass functions of the number of renewals and the renewal epochs for discrete renewal processes. Secondly, we put light on two interesting discrete renewal processes to model number of newly elected to at most two terms position, and performances of tennis players. In particular, we apply our derivations to model the US presidential election, and to model the performances of John Isner and Daniil Medvedev in Tennis R32 tournaments. In addition, we establish a formula for numerical evaluation of the k th visit in Markov chains and derive an alternative formula to the Chapman-Kolmogorov Equation.

Asymptotic behaviour of the hazard function from cross-sectional cohort survival data

A. Shariati

School of Mathematical and Physical Sciences, Macquarie University

To study the progress of a disease, a cross-sectional cohort study is more practical and feasible than an incident cohort design. In the former, cases who have experienced the initiating event, say the onset of disease, prior to the start of the study are recruited. The subjects are then followed-up for the remaining of their lifetimes until experiencing the terminating event, say death. The collection of data in such studies is subject to structural selection bias. The other challenge we encounter in all prospective cohort studies is loss to follow-up on some of the recruited subjects which leads to censored data. The survival data collected in prevalent cohort studies are commonly analysed by conditioning on the observed truncation times. Under stationarity which assumes that the initiating events follow a stationary Poisson process, we can, however, employ a more efficient strategy known as unconditional approach. We propose the unconditional nonparametric maximum likelihood estimator of the hazard function. We present results on the uniform strong consistency and weak convergence of the proposed estimator. Given the intractable form of the corresponding asymptotic stochastic integral, we propose a method to approximate the behaviour of the limiting process, which is then employed to obtain uniform confidence bands for the hazard function. The proposed method can be applied for other scenarios that the stationarity assumption does not hold if the intensity of the incidence process is known. The finite sample performance of our methodology is validated through simulation results. The procedures are applied to analyse censored survival data on patients with dementia collected in the Canadian Study of Health and Aging.

Fractional Brownian motion in financial markets and Greeks by Malliavin calculus

M. Tahmasbi

Department of Applied Mathematics, Tarbiat Modares University

Fractional Brownian motions are one of the most famous and applicable stochastic processes utilized in modeling of various physical and economical phenomenon and also in traffic and communication technology. In financial mathematics, these processes mostly are used to model volatility assets with both Hurst parameter more than half and also less than half. There are different concepts of stochastic fractional integral, such as Young integrals, Wikh integrals, Skorokhod integrals and rough path integrals. In this Lecture, we first give a brief introduction of the stochastic integrals driven by fractional brownian motion (fBm) and we state that how to price the derivatives in these models. Then, after considering path-dependent stochastic differential equations driven by fractional integrals, we apply Malliavin calculus approach to find a simple expression of Delta, which make its simulation easily capable, of the payoff functions especially for volatility derivatives.

Option valuation in markets with finite liquidity under fractional CEV assets

A. Ghasemifard

Faculty of Mathematical Sciences, University of Mazandaran

After the financial crisis, market traders realized that a better understanding of the limited liquidity influences on all features of the financial market was needed. One of the origins of such effects is the inclusion of the price impact of option hedging strategies resulting from the relaxation of the assumption of infinite liquidity of the market in underlying assets, which implies that trading affects the price of underlying assets, unlike in Black Scholes markets. Models that incorporate such an effect unavoidably lead to nonlinear feedback. In this talk, we numerically price the European double barrier option by calculating the governing fractional Black-Scholes equation in illiquid markets. Incorporating the price impact into the underlying asset dynamics, we consider markets with finite liquidity. We survey both cases of first-order feedback and full feedback. Asset evolution satisfies a stochastic differential equation with fractional noise, which is more realistic in markets with statistical dependence. Moreover, the Sinc-collocation method is used to price the option. Numerical experiments show that the results highly correspond to our expectation of illiquid markets.

Modified principal points

H. A. Madrani-fard

Department of Mathematics and Statistics, Yasouj University

The Modified Principal Points (MPP) is a modified version of the principal points introduced by Flury (1990). It provides a better distance function between a random vector X and a set of k points P . The new distance function is defined as the weighted average of distances between X and single elements of P , with weights determined by a kernel density function with mean zero. This distance function has two advantages: differentiability and flexibility with the variance of the kernel function. The MPP algorithm has been shown to improve optimization results compared to traditional principal points and has applications in clustering and other machine learning methods. The MPP algorithm has been tested on real-data and has demonstrated improved clustering results compared to traditional methods such as k- means and hierarchical methods.

Vlasov-McKean equations

S. Mehri

University of Warwick

We will review recent works on the Vlasov-McKean equations, stochastic law dependent equations. We state a propagation of chaos result for neural networks, which means solutions to a coupled stochastic differential equations with independent initial values converges to independent solutions to Vlasov-McKean equations.

Nonparametric statistical inference for i.i.d. sparsely observed diffusions: an FDA perspective

N. Mohammadi

Colorado State University

Functional Data Analysis (FDA) covers an undeniably central role in studying different statistical inference problems, allowing to consider functional datasets on possibly complex domains, with trajectories observed discretely or continuously. Concerning discrete observations, this approach basically imposes some smoothness conditions on the sample paths and/or their covariance function to apply well-developed approximating methods. However, the usual regularity assumptions seriously limit the appropriateness of FDA in many commonly encountered settings, most notably stochastic differential equations (SDE). In this talk, we introduce a careful modification of existing methods, dubbed the "reflected triangle estimator" and make inferences about the global behavior of the diffusion processes. We show that this allows for the FDA of processes with nowhere differentiable sample paths, even when these are discretely and noisily observed, including under irregular and sparse designs. We then proceed to relate the global behavior of the processes to their local behavior by means of an apparently novel PDE. We establish almost sure uniform asymptotic convergence rates of the proposed estimators as the number of observed curves grows to infinity. Our rates are non-asymptotic in the number of measurements per path, explicitly reflecting how different sampling frequencies might affect the speed of convergence.



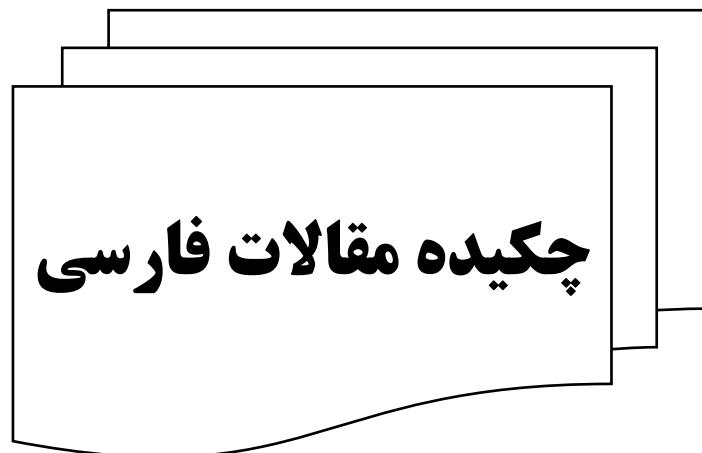
Local times of Gaussian random fields

Y. Xiao

Department of Statistics and Probability, Michigan State University.

We study the local times of anisotropic Gaussian random fields satisfying strong local nondeterminism with respect to an anisotropic metric. By applying moment estimates for local times, we prove optimal local and global Hölder conditions for the local times for these Gaussian random fields and deduce related sample path properties. These results are closely related to Chung's law of the iterated logarithm and the modulus of nondifferentiability of the Gaussian random fields.

We apply the results to systems of stochastic heat equations with additive Gaussian noise and determine the exact Hausdorff measure function for the level sets of the solution. This talk is based on a joint paper with Davar Khoshnevisan and Cheuk Yin Lee.



بیمه سبد سرمایه تحت مدل‌های رژیم-متغیر مارکوف

علی فروش باستانی^{*}، پیمان علیپور، غلامرضا منصورفر

دانشگاه تحصیلات تکمیلی علوم پایه زنجان

استراتژی‌های بیمه سبد^۱ روش‌هایی ساختارمند برای سرمایه‌گذاری هستند که با تعیین مقداری مشخص برای کف، سطح معینی از اطمینان (بیمه) به بازگشت سرمایه را ارائه می‌کنند. به بیان دیگر با استفاده از این استراتژی‌های معاملاتی می‌توان حداقل بازده از پیش تعیین شده ای را کسب کرد. در این پژوهش، ضمن بیان نحوه ساخت و تخصیص دارایی در این استراتژی‌ها، به بررسی عملکرد استراتژی بیمه سبد با نسبت ثابت^۲ و بیمه سبد بر اساس ارزش در معرض ریسک^۳ می‌پردازیم. در ابتدا مدل ریاضی رویکرد با نسبت ثابت در حالت محدود شده ارائه شده و در ادامه با استفاده از تبدیل فوریه تابع مشخصه، تابع چگالی بازده استخراج می‌شود. سپس با استفاده از تابع چگالی بازده، ارزش در معرض ریسک در سطوح اطمینان مدنظر به دست آمده و در نهایت به کمک آن، مدل ریاضی رویکرد مبتنی بر ارزش در معرض ریسک ارائه می‌شود. به منظور برآورد روند حرکتی دارایی ریسکی، از مدل با رژیم متغیر استفاده می‌شود که به واقعیت نزدیکتر است. نتایج نشان می‌دهد که هر دو استراتژی در کنترل ریسک نامطلوب موفق عمل کرده‌اند و این عملکرد با افزایش سطح اطمینان و تعداد دفعات تغییر چیدمان سبد در طول دوره سرمایه‌گذاری بهبود می‌یابد. در مقایسه دو استراتژی، استفاده از سنجه امگا نشان می‌دهد که در آستانه‌های پایین عملکردی، استراتژی با نسبت ثابت بهتر است. همچنین پراکندگی نتایج شبیه سازی شده برای ارزش نهایی سبد نشان می‌دهد که رویکرد با نسبت ثابت در محافظت از کف عملکرد بهتری دارد.

* سخنران اصلی

¹ Insurance Portfolio

² Constant Proportion Insurance Portfolio

³ Value-at-risk based Portfolio Insurance

روایت سفری پر ماجرا: گریزانی از احتمال تا علاقه مندی و افر به نظریه احتمال و فرایندهای تصادفی

دکتر بیژن زنگنه

گروه علوم ریاضی، دانشگاه صنعتی شریف

در این سخنرانی، سفر پر ماجرا احتمال‌دان شدنم را برایتان روایت می‌کنم. میخواهم راوی تجربه زیست‌های باشم که برای کسانی که مسئولیت تربیت ریاضی دانما را به عهده دارند، ارزش آموزشی دارد. در این سفر، استادان کم نظری حوصله کردند و گریزانی ام را با نشان دادن بدیل‌های آموزشی متناسب با سلیقه یادگیری ام، تبدیل به علاقه مندی زیادی کردند که زندگی حرفه‌ای ام را با عشق و شادابی، صرف آن کرده ام و همچنان از تدریس و پژوهش و یادگیری در این حوزه، به وجود می‌آیم.

رگرسیون لوثستیک فازی تحت خطاهای تصادفی فازی

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در این مقاله، مدل های رگرسیونی را مورد بحث و مطالعه قرار می دهیم که متغیرهای پاسخ به صورت دو ارزشی و متغیرهای توضیحی (پیشگو) اعداد معمولی هستند اما خطاهای علاوه بر ماهیت ابهامی، ماهیتی تصادفی نیز دارند. یعنی متغیرهای تصادفی فازی تعریف شده اند. بر این اساس مدل پیشنهادی را صورت بندی کرده و برآورد ضرایب را برای حالتی با تنها یک متغیر توضیحی و با استفاده از روش کمترین مربعات محاسبه می کنیم.

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